

Designing Stable Three Wheeled Vehicles,
With Application to Solar Powered Racing Cars
November 8, 2006 Revision

A Working Paper by:

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I. PREFACE TO 2006 REVISION

The original version of this working paper was written following Sunrayce '95 as a tutorial on the design of 3 wheeled vehicles and was sent to race organizers. It was revised in 1996 and again sent to race organizers in 1997 to inform the discussion of 3 wheeled and 4 wheeled vehicles. The current version is a further revision which includes a discussion of a stability indicator called the Understeer Gradient which can be measured in road testing of a completed vehicle. Reference to Sunrayce '95 is still included and the purpose is the same: to introduce some elementary concepts of vehicle dynamics and apply them to the design of stable 3 wheeled vehicles.

II. INTRODUCTION

Many of the vehicles that participate in solar car racing have three wheels, arranged with two in front and one at the rear. There were some incidents in Sunrayce '95 involving such vehicles, but also many of the top finishing cars were three wheelers. This suggests that there are probably some “do’s and don’ts” regarding the design of these cars. This paper will discuss the dynamics of three wheeled vehicles, and show how improved stability can be designed in.

A crucial vehicle property is the location of the vehicle center of gravity (CG). If it is located properly, the vehicle will be “stable” in terms of:

- a. Resistance to “losing the rear end” in turns and crosswinds.
- b. Ability to travel at high speed without continual steering corrections to counteract weaving.
- c. Resistance to tipping over in turns and in encountering changes in road surfaces if sliding.
- d. Resistance to swapping ends in hard braking due to weight transfer from the rear to front.

If the CG is in the “wrong place”, the vehicle may exhibit all these unstable behaviors.

During Sunrayce '95, I asked many advisors and students if they knew where the CG was on their cars. Very few knew. This is most disturbing. The location of the CG should be a design specification. Components should be arranged to achieve a specified location of the CG. It appears that few teams approach it this way. Rather, I suspect that components are arranged through an ad hoc process of fitting things where it is convenient or accessible or to solve some interference problem arising from previous ad hoc choices. Now that Sunraycers are capable of traveling at posted speed limits on sunny days, it is crucial that faculty and student designers understand how the location of the CG can influence vehicle stability.

This paper illustrates how stability can be designed into three wheeled vehicles through thoughtful choice of the CG location, both longitudinally and in height, the front track and the wheelbase. The approach employs the simplest models of vehicle dynamics and utilizes undergraduate level physics and mathematics. The simple models are not the complete story of vehicle response, but do capture the main factors of vehicle behavior. Specifically, there are no suspension systems in the models. Suspension will generally soften and delay the responses, but the tendency toward stable or unstable behavior will still be present.

The outcome will be the ability to state desirable vehicle responses in terms of yaw, tipping and braking weight transfer. There will follow a few inequalities that

involve the CG location, front track and wheelbase, allowing these values to be chosen to satisfy the desired responses. A major point is that the wheelbase, track and CG location should be treated as design decisions, to be specified, using the procedures described herein. Once specified, then components should be selected and located to achieve the desired CG value. The approach was used in the University of Minnesota Sunrayce'95 entry, Aurora II, which is used as an example.

III. CENTER OF GRAVITY LOCATION AND YAW RESPONSE

The yaw response of the vehicle refers its tendency to rotate about a vertical axis through the CG, or “spin”. A stable vehicle can undergo side loads as in cornering or wind gusts, and not suddenly yaw in such a way as to amplify the tendency to spin. It is possible to yaw slightly in a self-corrective manner. The type of response depends largely upon the location of the CG. The following sections will describe factors that connect CG location with yaw response. It first requires a brief description of how a pneumatic tire responds to side loads.

Tire Response to Side Load: Slip Angle

When a loaded rolling tire is subjected to a side load, its path is deflected from the direction in which the tire is headed. The angle of deflection is called the slip angle. Figure 1A shows a top view of a tire with side force \bar{F} applied at the axle line, with velocity V along the direction of travel, and slip angle α between the actual path and the tire heading. (It is presumed the axle rotates in bearings which keep the tire vertical to the road). Figure 1B shows the front view, with lateral force F acting upon the tire from the ground, and a vertical load N , along the centerline. Lateral force F is equal in magnitude to side force \bar{F} .

There is a relationship between the lateral force F and the slip angle, which is found experimentally and reported in the technical literature as plots of lateral force F vs. slip angle for various vertical loads. In that literature, the positive force direction is as indicated in Figure 1B, so force F had negative values for positive slip angle. Figure 2 shows such a plot for a 17” bicycle tire.

A tire has a maximum slip angle, above which the tire will slide. That is, if the side load \bar{F} exceeds the lateral load that the tire can produce at its maximum slip angle, then the tire will slide, rather than “slip”.

Figure 2 reports data for slip angles of ± 3 degrees, from which it may be implied that the maximum is not much above that value. Plots for automotive tires have maximum slip angles of around ± 10 degrees.

In literature for automotive enthusiasts, (Road and Track magazine, etc.) it is customary to plot the side force \bar{F} rather than the force acting upon the bottom of the tire, giving a plot in the first quadrant. Figure 3 shows such a plot for the data in Figure 2, in which Newtons are converted to pounds. As long as the tire is slipping, the magnitudes of \bar{F} and F are the same, so either plot can be used. We will use Figure 3. Three observations about Figure 3:

a) The lateral force developed by a tire at a certain slip angle will increase as the vertical load increases, but in a diminishing manner. For example, Figure 3 shows that at a 3 degree slip angle, a lateral force of 37 Lb. corresponds to a load of 67 Lb. while, doubling the load to 135 Lb. only increases the lateral load to 56 Lb. or a 66% increase.

FIGURE 1A: TOP VIEW

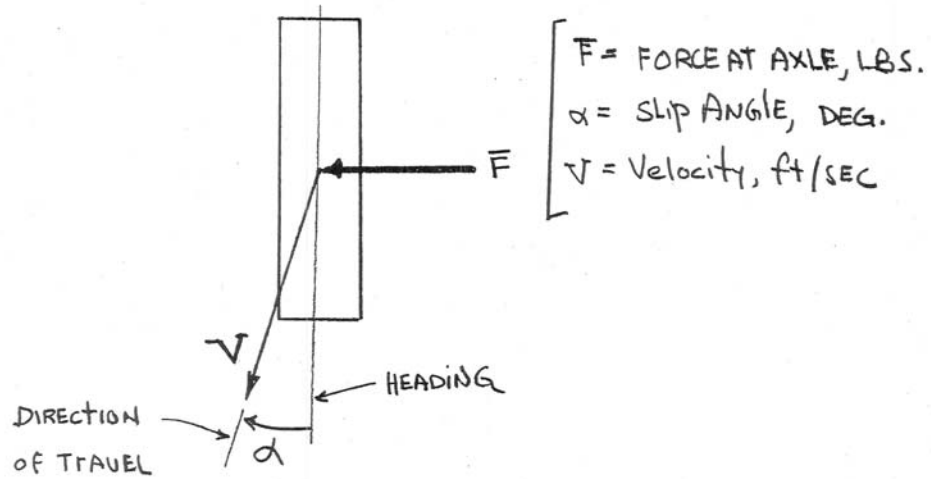
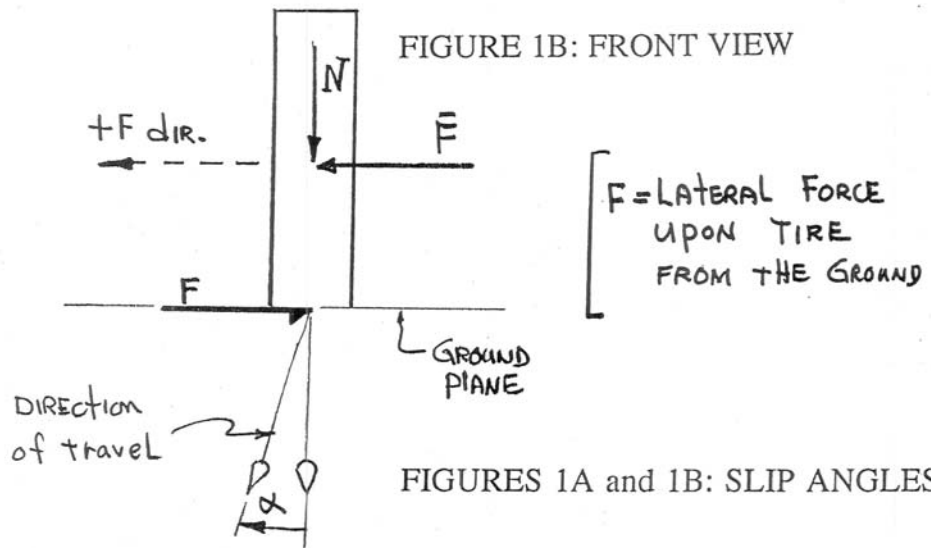


FIGURE 1B: FRONT VIEW



FIGURES 1A and 1B: SLIP ANGLES

FROM: LECTURE 3-3: SUNRAYCER WHEELS, TIRES AND BRAKES,
CHESTER KYLE, GM REPORT

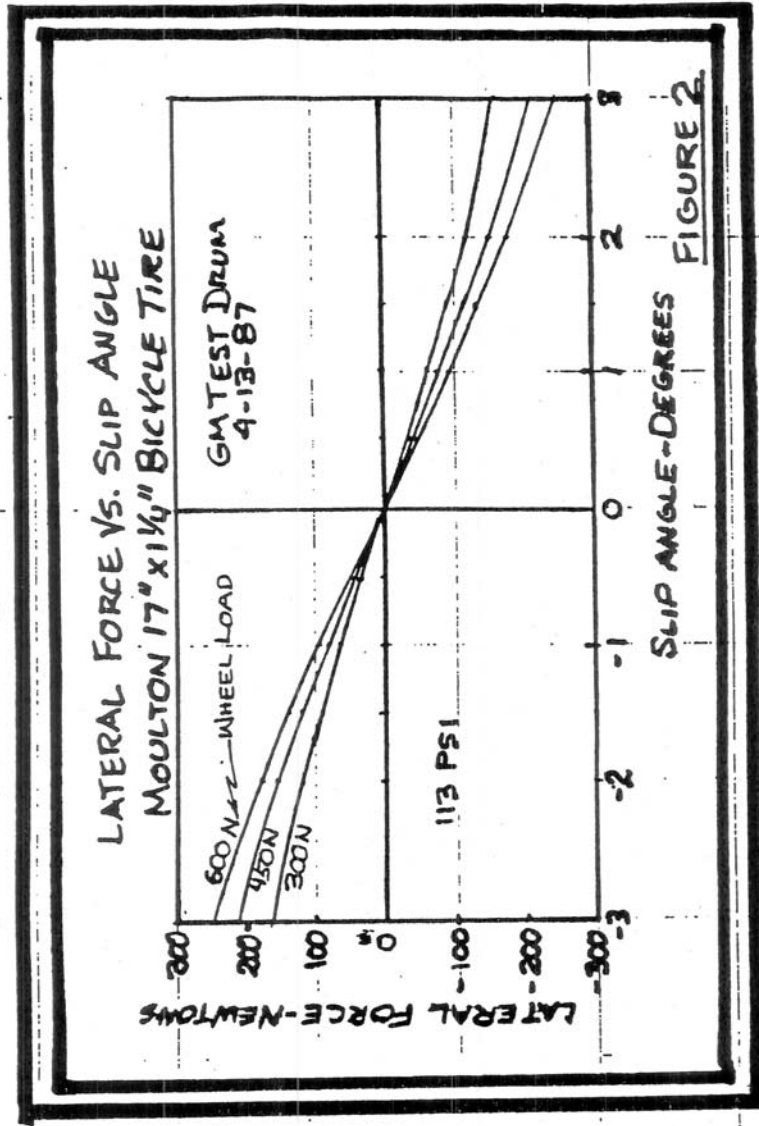


FIGURE 2: LATERAL FORCE vs. SLIP ANGLES

This diminishing of the change in lateral force per unit change in load is referred to as “load sensitivity”.

b) The curves grow from the origin in a linear manner, and the slopes decrease as the slip angle increases. The literature uses the symbol “C” to describe the slope in the linear range and it is called “(tire) cornering stiffness”, and has the units of Lbs. of lateral force per degree of slip angle. The value of C also exhibits load sensitivity. Consider Figure 3 with load N=101 Lb. as “nominal”, having C=17.8 Lb./degree. Decreasing the vertical load by 34 Lb. reduces C to 13.5 Lb./degree, or a change of 4.3 Lb./degree, but increasing the vertical load by 34 Lbs. increases C to 20.7 Lb./degree or a change of 2.9 Lb./degree. As load increases, the value of C increases in a diminishing manner.

c) A “coefficient of lateral friction” can be interpreted as the ratio of the lateral force divided by the vertical load. Considering the 3 degree slip angle and 67 Lb. vertical load, the lateral force is 37 Lb. Thus the coefficient of lateral friction is:

$$\mu_c = \frac{37}{67} = 0.55 \quad (1)$$

At larger vertical loads, this value would be smaller, due to the diminishing increase in lateral loads.

Figure 3 data is for a tire that presumably supports a maximum of 135 Lbs. Three of these would be too small for a typical sunraycer (with lead-acid batteries) of about 800-900 Lbs. For Sunrayce '97 and later events, organizers required a DOT rated tire and some manufacturers have developed low rolling resistance tires specifically for solar and high mileage experimental vehicles. I do not have the “Figure 3” type of data for such tires, but in the following, it will be shown that while the concept of tire cornering stiffness is needed to develop relationships between CG location and stability, the actual values of cornering stiffness will not be needed. Topics will be introduced in terms of a four wheeled vehicle, two on the front axle line which provide steering, and two on the rear axle line. Then the results will be adapted to three wheeled vehicles

Vehicle Response to Side Loads at the CG: Static Margin

Figure 4 shows the top views of a “bicycle model” of the four wheeled vehicle in three conditions. In the bicycle model, each end of the vehicle has a single tire which is presumed to have the double cornering stiffness of the actual tires at each end of the four wheel vehicle. The CG is assumed to be on the center line of the vehicle. We are interested in the horizontal forces which act upon the CG and the tire contact patches, and the bicycle is constrained so as not to fall over (!). Three positions of the center of gravity are considered, denoted as CG, CG1 and CG2. Figure 4A shows the vehicle headed straight along path P, traveling at steady velocity V. The velocities of the axle line above the front and rear tire contact patches are indicated as VF and VR, and have the same magnitude and direction as V. Forces which are not shown are the driving thrust at the rear (or front) wheel, the aerodynamic drag and the rolling resistance, which are assumed to be along the path P and sum to zero.

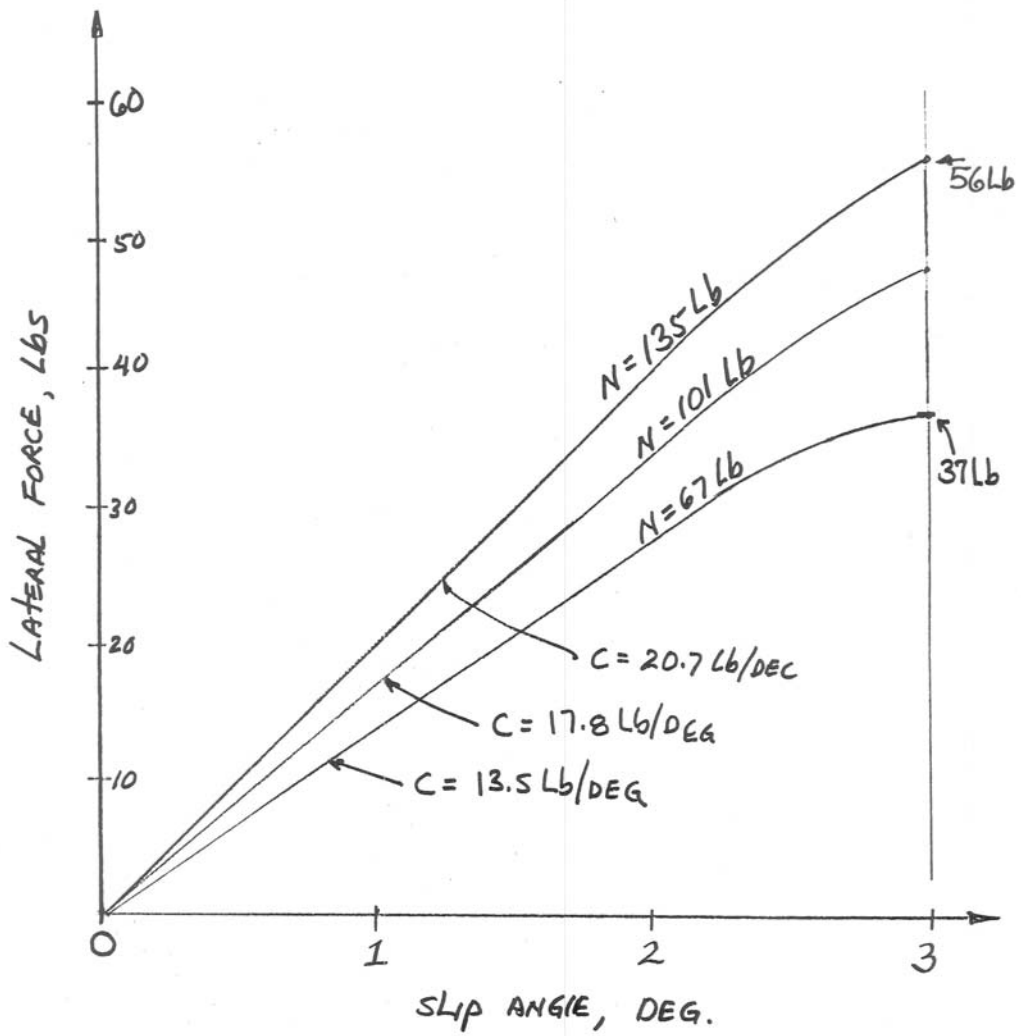


FIGURE 3: REPLOTTING FIGURE 2 DATA AND CONVERT NEWTONS TO LBS.

Figures 4B and 4C show side load FS applied at each center of mass and the resulting responses, presuming the driver makes no steering corrections. Figure 4B shows the vehicle traveling along path P at the instant of the application of the side load. Lateral forces at the tires, denoted as FF and FR, would develop to counteract the side load. Corresponding slip angles α_F and α_R would be present, as the velocities veer off of their original paths. Figure 4C shows three vehicle responses a short time after the application of the side force, where the tire forces and slip angles have been removed for clarity. The response is a sideways movement of the CG off the path P and an angular rotation about the CG, which is the yaw response and indicated by angle Θ . Also shown is a “Neutral Steer Point” (NSP). This is a point at which a side load can be applied and not cause a yaw response. The location of the NSP depends upon the total cornering stiffness values at each end of the vehicle. The distance from the front axle line to the NSP is:

$$LNSP = \left[\frac{\bar{C}_R}{\bar{C}_F + \bar{C}_R} \right] WB, \text{ ft.} \quad (2)$$

Where: C_F, C_R = cornering stiffness values of a single front and rear tire, Lbs/degree.

\bar{C}_F, \bar{C}_R = total cornering stiffness values for the four wheeler bicycle model:

$\bar{C}_F = 2 C_F$, Lbs/degree

$\bar{C}_R = 2 C_R$, Lbs/degree

WB = wheel base, ft.

LG = distance from front axle line to CG, ft.

Static Margin

The location of the NSP relative to the CG determines the character of the yaw response. The distance from the CG rearward to the NSP divided by the wheelbase is termed “static margin” (SM), or as measured from the front axle line:

$$SM = \left[\frac{\bar{C}_R}{\bar{C}_F + \bar{C}_R} - \frac{LG}{WB} \right] \quad (3)$$

The value of SM can be positive, negative or zero and its value is an indicator of the yaw response of the vehicle. To illustrate, we will examine the special case when the center of mass is “close” to one half of the wheelbase from the front axle and the same types of tires are used at each end. In that case, $\bar{C}_R \approx \bar{C}_F$ and equation (3) can be rewritten as:

$$SM = \left[\frac{1}{2} - \frac{LG}{WB} \right] \quad (4)$$

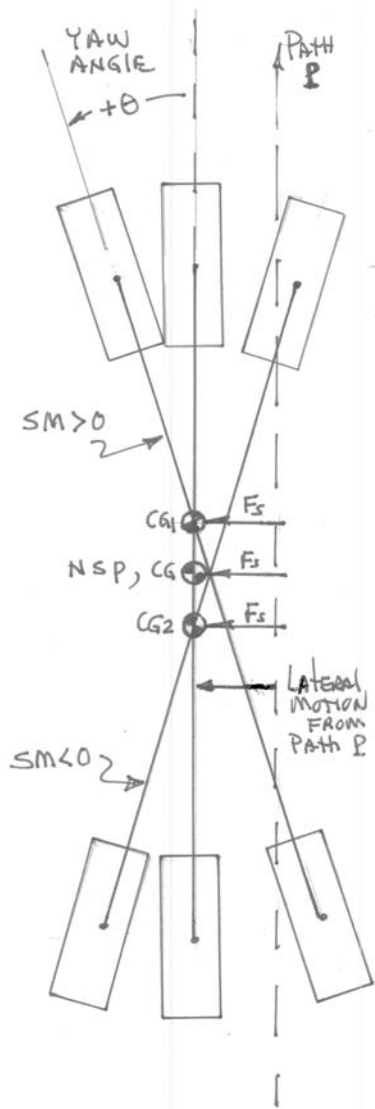


FIG. 4C:
RESPONSES TO SIDE LOAD

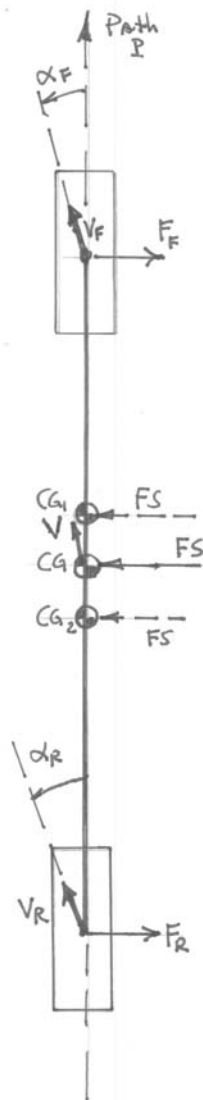


FIG. 4B:
APPLY SIDE LOAD

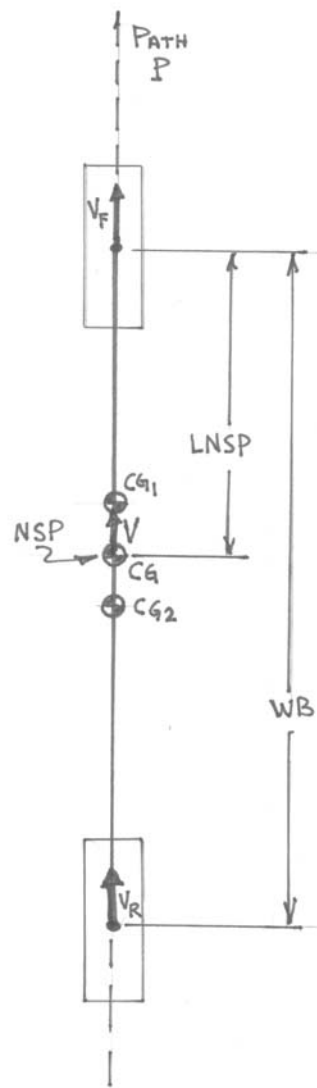


FIG. 4A
STRAIGHT AHEAD

FIGURE 4. BICYCLE MODEL WITH SIDE LOAD

The following observations apply to both the equations (3) and (4) forms, but are more easily “seen” in the equation (4) form.

$SM = 0$. When the center of mass is exactly at one half of the wheelbase, $LG = \frac{WB}{2}$ and $SM = 0$. In that case, a load at the CG will not cause any yaw response. The vehicle will simply sideslip, with front and rear slip angle being equal as illustrated in Figure 4C. It is termed “Neutral Steer”.

$SM = (+)$. When the CG is ahead of the NSP, shown as CG1, then LG/WB is less than one half and SM is positive. Yaw response is shown in Figure 4C in which the front slip angle is larger than the rear, and the vehicle is headed off in the direction of the applied force. This is considered “stable”, (for reasons to be discussed shortly), and is termed “understeer”.

$SM = (-)$. When the CG is behind NSP, shown as CG2, then LG/WB is larger than one half and SM is negative. Yaw response is shown in Figure 4C in which the rear slip angle is larger than the front, and the vehicle is turned against the direction of the applied force. This is considered “unstable” (to be discussed shortly) and is termed “oversteer”.

Typical vehicles have small positive values of SM , in the range +0.05 to +0.07 [2 p.209]. The seminal work of [3] reports $SM=+0.06$ for an American passenger car, and +0.003 for a Ferrari Monza sport racing car.

The previous suggests that positive or zero values of SM are desirable. The following will expand this discussion by considering the vehicle in a turn, where the “side force” will now be the centrifugal force.

Vehicle Response in a Turn

Figure 5 shows the vehicle in turn of radius R and traveling at velocity V . Now a steering angle δ is present. The velocities of the front and the rear axles are indicated as V_F and V_R , and corresponding slip angles are indicated. The driver has turned the steering wheel, causing the front wheel to turn angle δ from its straight ahead position, and negotiating a turn of radius R . Here, the “side load” is the centrifugal force, which acts upon the CG and increases with the square of velocity V and decreases as the radius R increases. The wheelbase is considered small compared to R , and all angles are considered small. In that case, the steering angle in degrees can be expressed as follows, from the geometry of Figure 5.

$$\delta = (57.3) \frac{WB}{R} + (\alpha_F - \alpha_R), \text{ degrees} \quad (5)$$

Angle β , called the sideslip angle, indicates the angle between the vehicle centerline and the direction of the velocity of the CG.

The steering angle δ of equation (5) can be expressed in ways which provide another connection between the CG location, tire cornering, stiffness, and stability. By writing force and torque balances for the vehicle of Figure 5, following [2, p202], and utilizing lateral force vs. slip angle data such as Figure 3, equation (5) becomes:

$$\delta = 57.3 \frac{WB}{R} + \left[\frac{W_F}{\bar{C}_F} - \frac{W_R}{\bar{C}_R} \right] \frac{V^2}{gR} \quad (6)$$

The V^2/gR term is the lateral acceleration in terms of the fraction of g's, and will be abbreviated as:

$$a_y = V^2/gR, \text{ lateral acc., g's} \quad (7)$$

Where V = velocity of CG along the path, ft./sec

R = radius of the CG path, ft.

g = acceleration of gravity, 32.2 ft./sec^2

The term in brackets has special significance and is called the “understeer gradient” and denoted with symbol K.

$$K = \left[\frac{W_F}{\bar{C}_F} - \frac{W_R}{\bar{C}_R} \right], \text{ understeer gradient, degrees/lateral g} \quad (8)$$

Where W_F = weight on front wheels, Lbs.

W_R = weight on rear wheels, Lbs.

\bar{C}_F = total front cornering stiffness, Lbs./deg

\bar{C}_R = total rear cornering stiffness, Lbs./deg

Then equation (6) can be abbreviated as:

$$\delta = 57.3 \frac{WB}{R} + (K)(a_y) \quad (9)$$

The steering angle can also be expressed in terms of the static margin by combining equations (3) and (6) to yield:

$$\delta = 57.3 \frac{WB}{R} + (W) \left[\frac{\bar{C}_F + \bar{C}_R}{\bar{C}_F \bar{C}_R} \right] (SM) a_y \quad (10)$$

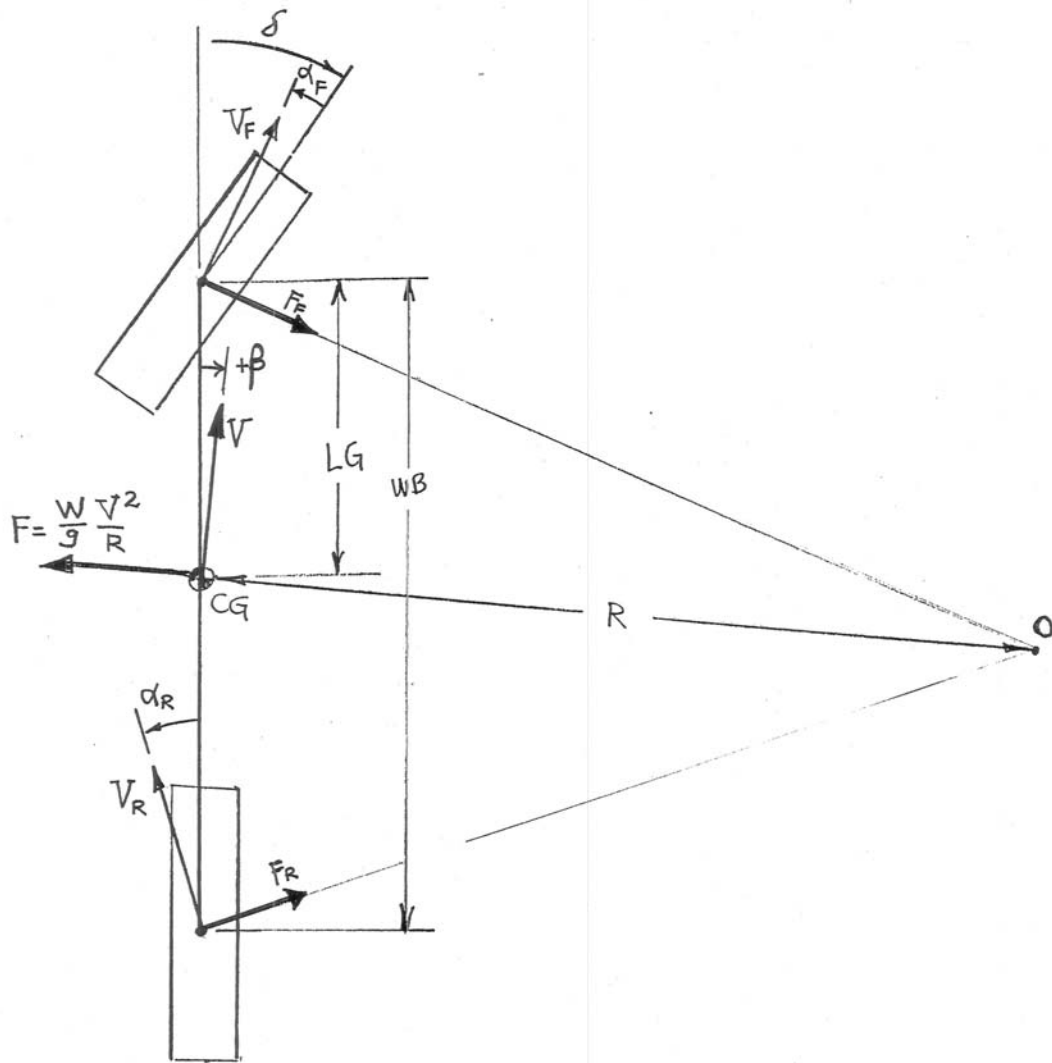


FIGURE 5: BICYCLE MODEL IN A TURN OF RADIUS R AT VELOCITY V AND STEERING ANGLE δ

To analyze stability in a turn, we are not interested in the exact value of steering angle δ , but rather how δ changes as the lateral acceleration, a_y , increases. Each expression for δ is the sum of a constant term, plus an expression involving one of the following terms, which could be positive, zero, or negative.

From Equation (5): $(\alpha_F - \alpha_R)$

From Equations (8, 9): $K = \left[\frac{W_F}{C_F} - \frac{W_R}{C_R} \right]$

From Equations (10, 3): $SM = \left[\frac{\bar{C}_R}{\bar{C}_R + \bar{C}_F} - \frac{LG}{WB} \right]$

The conditions that change the signs of these terms are equivalent: for example, when $\alpha_F > \alpha_R$, then $K > 0$ and $SM > 0$. It is the sign change of these terms that determines stability and defines the vehicle steer characteristics.

Neutral Steer: $\alpha_F = \alpha_R, K = 0$ and $SM = 0$.

Steering angle δ remains constant at the value $57.3 \frac{WB}{R}$, degrees. If the vehicle was negotiating a circular path of radius R and slowly increasing the velocity, causing a_y to increase, more lateral force would be needed at each end of the vehicle which would require slip angles α_F and α_R to increase. For a neutral steer vehicle, the vehicle centerline will slightly rotate relative to the direction of velocity V, decreasing the side slip angle β (which may become negative) and thus increase both front and rear slip angles the same amount as δ remains constant.

Understeer $\alpha_F > \alpha_R, K > 0$ and $SM > 0$

The needed steering angle increases with speed, as the front slip angle is larger than the rear. However, it is self correcting. That is, if the driver does not increase δ , the positive yaw will turn the front of the vehicle toward the outside of the original path, thereby increasing the radius R and reducing the centrifugal force. Thus, a positive SM is termed “stable”.

Oversteer: $\alpha_R > \alpha_F, K < 0$ and $SM < 0$

The needed steering angle is reduced from its neutral steer value, since the rear slip angle is larger than the front. That is, the rear is side slipping more than the front. If the driver makes no correction, the rear of the vehicle moves outward from the original path, decreasing the radius R and increasing the centrifugal force, requiring a greater correction. The correction is the familiar “turn in the direction of the skid”, and if done too late, one may not be able to “catch” the vehicle before it spins. Thus a negative SM is termed “unstable”.

Section IV describes a driving test that can estimate the Understeer Gradient for a vehicle.

Design Implications for Four Wheelers

On the basis of the bicycle model of a vehicle subjected to a side load at the CG, such as negotiating a turn at a steady speed, a stable response can be insured if the Understeer Gradient, $K \geq 0$ or equivalently the static margin, $SM \geq 0$. From the design view point, the values of K and SM depend upon the fore-aft location of the CG and upon the tire stiffness values. However, if the same tire is used at each location and they are equally loaded, then the total stiffness values at each end would be the same, $\bar{C}_F = \bar{C}_R$. Equations (8) and (3) would yield $K=0$ and $SM=0$, and the weight distribution for this four wheeler would be:

$$W_F = W_R = \frac{W}{2} \quad \text{and} \quad LG = \frac{WB}{2} \quad (11)$$

This corresponds to the 50/50 weight distribution. As the CG is moved further forward with the same tires at each location, the values of K and SM would become positive.

Design Implications for Three Wheelers

To utilize the bicycle model for a three wheeled layout, two wheels must be on the same axle line and placed laterally equidistant from the fore-aft vehicle centerline, and the third tire placed on that centerline. Consider the two wheels to be at the front, providing steering. The expressions for K and SM must be adjusted appropriately, but the $K \geq 0$ and $SM \geq 0$ conditions for stability do not change. The terms in the expressions for K and SM that need adjusting are the tire cornering stiffness values. Recall that C_F and C_R represent the stiffness values of single tires and that the \bar{C}_F and \bar{C}_R represent total stiffness values at front and rear. For the four wheeler, the C_F and C_R were simply doubled to provide the \bar{C}_F and \bar{C}_R values. For the three wheeler with two wheels at the front, the expressions for K and SM can be written in terms of the single tire stiffness values as:

$$K = \left[\frac{W_F}{2C_F} - \frac{W_R}{C_R} \right] \quad (12)$$

$$SM = \left[\frac{C_R}{2C_F + C_R} - \frac{LG}{WB} \right]$$

The threshold values of $K=0$ and $SM=0$ can be obtained by utilizing the same type of tire at each location and having equal weight on each tire. Then each tire would have the same stiffness values, $C_F = C_R$ and the corresponding weight distribution for this three wheeler would be:

$$W_F = \frac{2W}{3}, \quad W_R = \frac{W}{3} \quad \text{and} \quad LG = \frac{WB}{3} \quad (13)$$

The W_F is the load on two front wheels, so the load on each wheel is $\frac{W}{3}$. As the CG moves forward from this location, then both K and SM would become positive.

IV. CONSTANT RADIUS TEST / CRITICAL VELOCITY / THREE WHEELERS

This is a driving test to experimentally determine the value of K, the Understeer Gradient [2, p.227]. Equation (5), repeated below, shows how steering angle δ varies as a_y , the fraction of lateral g acceleration.

$$\delta = 57.3 \frac{WB}{R} + [K]a_y \quad (14)$$

Figure 6 shows how this equation would plot, where the slopes of the straight lines are values of K, which could be positive, zero or negative. Note that if K is negative, there is a value of a_y (and hence some velocity) for which angle δ becomes zero. This is called the “critical velocity” and is expressed as:

$$V_c = \left[\frac{57.3(WB)(g)}{K} \right]^{\frac{1}{2}}, \text{ critical velocity ft/sec (with K = negative)} \quad (15)$$

The vehicle is directionally unstable at V_c . The computation of V_c requires the numerical value of K, so tire cornering stiffness data would be needed. Note that V_c will increase with the wheelbase, enhancing stability.

The Constant Radius Test consists of driving in a circle of radius R at a steady speed and recording the steering angle and speed. Then incrementally increasing the speed and repeating the measurements until a specified maximum lateral a_y value is attained, or the vehicle is unable to maintain its path. The specified maximum lateral a_y value would be that which corresponds to the “tipping threshold”, discussed in the next section. Steering angle measurement could be at a vehicle wheel or the steering wheel and the speed could be estimated from a “lap time” around the circle where the driver is trying to maintain a steady speed.

Plot A on figure 8 shows hypothetical test data for a vehicle which would have a computed $K > 0$. The actual value of K would be estimated from the slope of a line through the apparently linearly increasing data points at low a_y values, but as a_y increases, steering angle δ increases non-linearly until a limiting value is reached. This occurs at a speed and corresponding a_y value where the needed lateral tire forces exceed their capability and the vehicle cannot maintain the path. In plot A, the limiting a_y value for the test vehicle is shown to exceed the specified maximum value.

The plot A data has a positive slope and hence $K > 0$ through the vehicle's entire range of cornering capability. It understeers at low a_y values and has limit understeer at high a_y values. This is considered a STABLE response.

The non-linear increase in δ implies that the value of K is increasing as a_y increases. This can be partially explained by the method the bicycle model uses to represent cornering stiffness. That is, the expression for K , equation (8), employs the cornering stiffness values which are presumed constant for small slip angles and a given load. However, as a_y increases, the lateral tire forces must increase, which causes the lateral force vs. slip angle relationship (such as figure 3) to move beyond the linear range. Also, lateral weight transfer from the inside tires to the outside, which occurs in an actual vehicle, would introduce load sensitivity effects so the total cornering stiffness of two tires on an axle would be less than the sum of their statically loaded values as was assumed in the bicycle model.

For a three wheeled vehicle having two wheels at the front, this lateral weight transfer has special significance and design implications. For such a vehicle, only the front two wheels will experience lateral weight transfer and so only the total front stiffness value would decrease from its presumed $2C_F$ value due to load sensitivity. Equation (12) shows that this decrease in total front stiffness will increase the K value as a_y increases.

The design implication is that the computed value of K could be slightly negative, based upon the CG location and cornering stiffness data, but as lateral acceleration a_y increased, the value of K would become positive, indicating a stable vehicle. For this reason, the specification for weight distribution for three wheeled vehicles (with two at the front) can be relaxed from a maximum of $33\frac{1}{3}\%$ of the weight on the rear, to a larger value, such as 36%.

V. TIPPING OF THE THREE WHEELED VEHICLE

Another form of vehicle stability is the resistance to tipping over in a turn. The simplest model to examine this situation is the "quasi-static" model which identifies the level of lateral acceleration, a_y , that causes the inside tire(s) to have zero vertical load. That level of lateral acceleration is called the "tipping threshold" and will be denoted by symbol F_c . For a four wheeled vehicle, the value is easily determined [2, p.310] as:

$$F_c = \frac{TR}{2HG}, \text{ four wheel tipping threshold, lateral g's.} \quad (16)$$

Where: HR = height of CG

TR = track (assuming front and rear are about equal)

The units of HG and TR must be consistent.

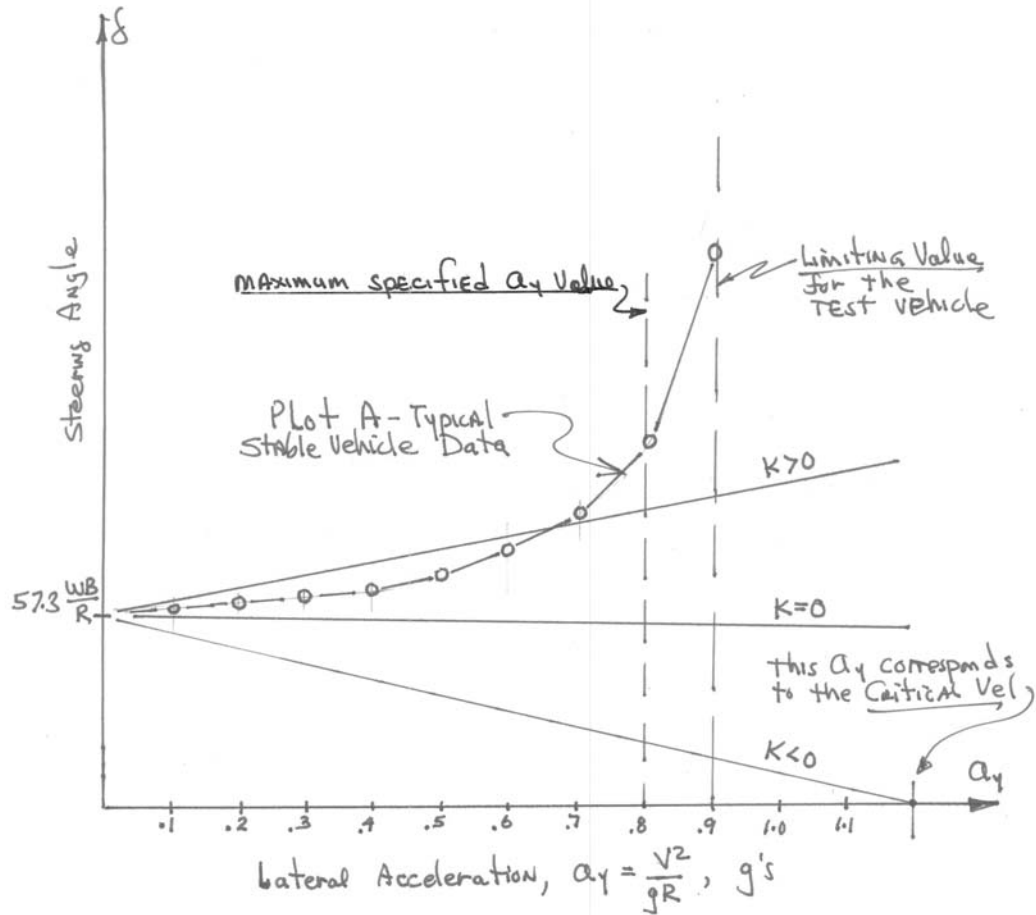


FIGURE 6 Plot of STEERING ANGLE δ vs a_y FOR EQN (14) AND TYPICAL EXPERIMENTAL DATA

The value of F_c for a three wheeler is slightly more complex and will involve the longitudinal placement of the CG as well as its height, and the vehicle track [5, 6]. Figure 7 shows a three wheeled vehicle in a right turn with vertical reactions at the tire contact points and lateral forces represented as lateral friction coefficients multiplied by their vertical loads. The side load at the CG is stated as fraction of the acceleration of gravity, a_y multiplied by the vehicle weight, W .

The vehicle is shown in a right hand turn, with subscript “i” denoting the inside front wheel, and “o” denotes the outside. The height of the CG is denoted as the HG and the front track is TR, with LG being the distance from the front axle rearward to the CG as defined previously. The CG is on the fore-aft longitudinal centerline.

There will be some value of the side load at which the vehicle will tip over. This may not actually occur, however, since the car may slide before it tips over. In fact the largest value of a_y that the vehicle will experience is equal to the coefficient of lateral friction at the tires, which for the bicycle tire of Figure 3 was seen to be about 0.55. However, it is prudent to design the vehicle for a higher lever of a_y since tires can encounter bumps, or changes in the road or shoulder surfaces while sliding. To implement this in the analysis, it is assumed that the tires are “riding on rails”, or constrained so as not to move sideways. The lateral forces are simply whatever is needed to keep the vehicle from sliding, so “tipping” can be examined. This will occur when the inside front wheel leaves the ground, or when $WF_i = 0$. At this point, the vehicle is supported by the outside front wheel and rear wheel. By applying simple statics to Figure 7 in this condition, the following inequality can be developed:

$$a_y \geq \frac{TR(WB - LG)}{2(WB)(HG)} = F_c \quad (17)$$

F_c is the cornering “g” level at which tipping is initiated and bigger is better. Expression (17) could be used as a performance specification in the initial design stage. A minimum value of F_c can be selected and its effect upon the options for track, wheelbase and CG location can be explored. The expression shows that increasing the front track, decreasing LG and lowering the CG all contribute to a larger F_c value. In the Aurora II, we used $F_c = 1$ as a performance specification. It can be shown that the value of F_c is related to the angle of a “tipping table” at which the inside wheel leaves the ground as follows:

$$\text{TAN (tip angle)} = F_c \quad (18)$$

Sunrayce used a tipping table in 1990 with a 35° angle corresponding to an F_c value of 0.7.

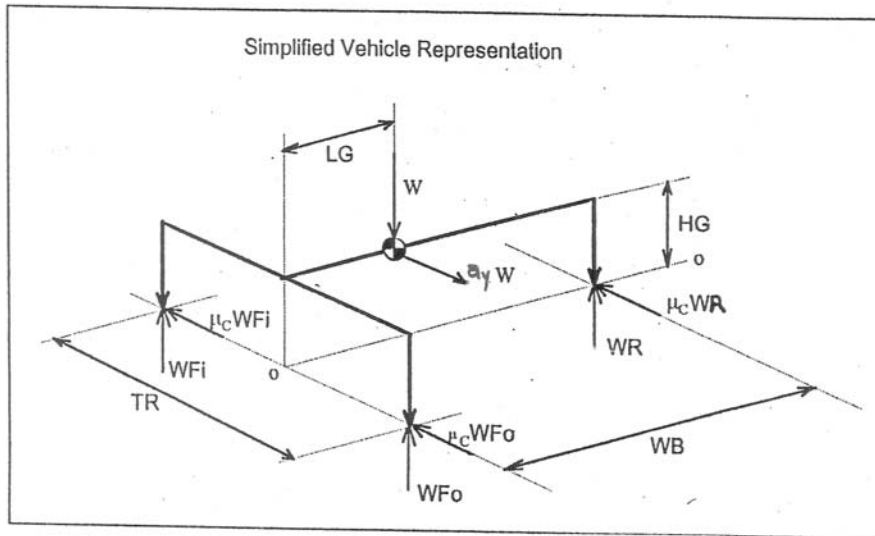


FIGURE 7. FORCE DIAGRAM FOR TIPPING ANALYSIS, RIGHT HAND TURN

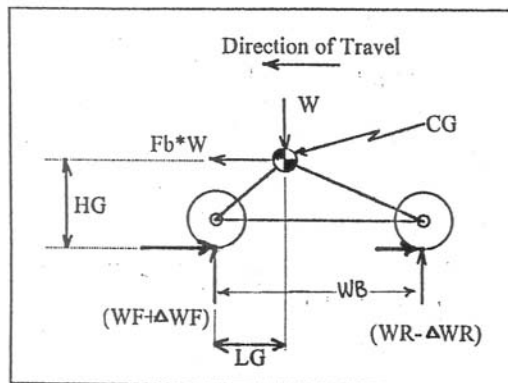


FIGURE 8. FORCE DIAGRAM FOR BRAKING WEIGHT TRANSFER

VI. WEIGHT TRANSFER IN BRAKING

For a three wheeled vehicle that has 2/3 or more of the weight on the front two wheels and a high CG, one could envision that the forward weight transfer under hard braking may severely reduce the vertical load at the rear. This could be studied as a potential yaw situation in vehicle dynamics, but in keeping with the use of simple models, a quasi-static approach will be used to determine the fraction of the static weight at the rear that is transferred to the front in braking. Figure 8 shows a side view of a vehicle under braking. It could have 3 or 4 wheels. The braking deceleration is denoted by F_B as a fraction of the acceleration of gravity. The static vertical load at the front is W_F and on the rear is W_R . Weight transferred to the front during braking is denoted as ΔW_R , which is a positive value. The weight transfer from the rear to the front, expressed as a fraction of the initial weight on the rear is:

$$\frac{\Delta W_R}{W_R} = \frac{(HG)(F_B)}{LG} \leq F_T \quad (19)$$

Where: ΔW_R = weight transferred under braking, Lbs.

W_R = static weight on rear, Lbs.

F_B = braking deceleration, g's.

F_T = target value

Equation (19) is performance measure and a target value, denoted as F_T could be set at the initial design stage. Lower values of HG and larger values of LG, that is a more rearward location of the CG will give less weight transfer, thus this target can conflict with the tipping specification. Braking requirements for the Sunrayce and ASC events required an F_B value of about 0.5g. For the Aurora II vehicle, we used $F_T=0.3$ and $F_B=0.5$. The value of F_T is more towards the “useful information” category, in contrast to the Understeer Gradient and tipping threshold which are serious determinants of stability.

VII. CONCLUSIONS

This paper presented a tutorial treatment of elementary vehicle dynamics models in order to show how certain vehicle parameters affect vehicle stability. The concepts of slip angle, cornering stiffness, yaw response, neutral steer point, Static Margin (SM), Understeer Gradient (K) and tipping threshold were described. The “bicycle model” was introduced and its yaw response was described when subjected to both a side load while running in a straight line, and to a centrifugal force while cornering. Vehicles with $K \geq 0$ or $SM \geq 0$ are considered to have a stable yaw response. The expressions for K and SM involve the fore-aft location of the vehicle CG, expressed as distance LG or equivalently W_F and W_R , and the tire cornering stiffness values. These stiffness values may not be available data, but if the same type of tire is used at each location, then the threshold conditions of $K=0$, $SM=0$ can be achieved by choosing the fore-aft location of the CG as follows:

4 Wheeler, $LG = \frac{WB}{2}$ giving 50%/50% front/rear weight distribution

$\frac{3 \text{ Wheeler}}{(2 \text{ at front})}$, $LG = \frac{WB}{3}$, giving 66%/33% front/rear weight distribution

For both of these layouts, as the CG moves further forward, then $K > 0$ and $SM > 0$, insuring understeer. For solar vehicles, tires are generally selected for their low rolling resistance properties, so the same tire is usually used at each location.

Expressions were presented that identified the level of lateral g 's that would initiate tipping of both a four wheeled and three wheeled vehicle. The expression for the latter involved the CG height, the track and the fore-aft location of the CG. A "tipping threshold" value, denoted as F_c , was discussed as a means of constraining the choice of the CG location and track so all vehicles would meet a minimum standard of tipping stability, which could be verified with a tipping table.

The Constant Radius Test was described as a means of experimentally determining K . A typical experimentally determined plot of δ vs. a_y showed that a stable vehicle will have K increasing as a_y increases, and this phenomenon was attributed to the way the total cornering stiffnesses are affected by weight transfer in an actual vehicle. For a three wheeled vehicle with two at the front, one would expect the value of K to increase as a_y increases, which allows room to slightly relax the weight requirement of 33% on the rear to perhaps 36%. However, rather than discuss how much to relax the rear weight distribution, a better way would be to simply require all teams to perform the Constant Radius Test as a means to verify the stability of all race entries. Full scale vehicle tests are performed as part of the scrutineering at solar vehicle competitions in the USA, and they include a braking test and negotiating a figure eight course. "Stability" is informally determined by observers watching how the vehicles respond. Rather than administer the test, race organizers could require teams to provide a report which would include an experimentally determined plot of steering angle δ vs. a_y as shown in figure (6), for a_y values up to the tipping threshold that would be specified in the rules and tested with a tipping table at scrutineering. It would be required that the slope of the δ vs. a_y curve indicates that $K \geq 0$ for low a_y values, and that as a_y increases, then K maintains a positive value. Steering angle data could be easily taken from a steering column with a string potentiometer and vehicle speed should already be available. Vehicles having K values (slopes) that become negative as a_y increases would not be allowed to run. A test report would be submitted by each team and signed as authentic by the advisor. This would raise the awareness of vehicle dynamics by all teams and provide an actual MEASURE of vehicle stability.

VIII. REFERENCES

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